ASSIGNMENT SET – I

Mathematics: Semester-II

M.Sc (CBCS)

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

PAPER - MTM-203

Paper: Algebra

1. Express the matrix $\begin{pmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \\ -1 & -1 & 4 \end{pmatrix}$ in Jordon Canonical form.

2. For two matrices A and B show that AB and BA need not have the same minimal polynomial.
3. Let A be an n × n matrix. (a) Suppose that A^2 = O. Prove that A is not invertible. (b) Suppose that AB = O for some nonzero n × n matrix B. Could A be invertible? Explain.

4. Let V and W be finite-dimensional vector spaces, and let T: V → W be a linear transformation. Suppose that β is a basis for V. Prove that T is an isomorphism if and only if T(β) is a basis for W
5. Let V be an inner product space, and let T be a linear operator on V. Then T is an orthogonal projection if and only if T has an adjoint $T^*$and T^2= T = $T.^*$
6. Define $T: P(R) \to P(R)$ by $T\big(f(x)\big) = \int_0^x f(t)$ Prove that T linear and one-to-one, but not onto.
7. Give an example of a linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ such that N (T) = R(T).

8. Let v be a finite dimensional vector space , and define $\psi: V \to V^{**}$ by $\psi(x) = \bar{x}$ then prove that ψ is an isomorphism.
9. State and prove Sylvester's Law of Inertia on finite dimensional real vector space V.
10. Let T be the linear operator on $P_2(\mathbb{R})$ defined by $T\big(g(x)\big) = -g(x) - g'(x)$ then find a Jordan Canonical form Of T and a Jordan canonical basis for T.
11. Find the Minimal polynomial of the matrix $\begin{pmatrix} 3 & -1 & -0 \\ 0 & 2 & 0 \\ 1 & -1 & 2 \end{pmatrix}$
12. Define Bilinear form on vector space V over a field F
13. Find the Characteristic polynomial of the companion matrix
14. Give an example a complex symmetric matrices need not be normal
15. Give an example of a linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ such that N (T) = R(T)
16. Let V be the vector space of all polynomial function p into $\mathbb{R}$ which have degree 2 or less .Define three functions on V given by $f_1(p) = \int_0^1 p(x)dx$ $f_2(p) = \int_0^2 p(x)dx$, $f_3(p) = \int_0^{-1} p(x)dx$ show that { f_1, f_2 f_3} is a basis of V^*
17. Show that every subspace of V is invariant under I and θ the identity and zero operators.
18. Show that two nilpotent matrices of order 3 are similar if and only if they have the same index of Nilpotency . Show by example that the statement is not true for nilpotent matrices of order 4.
19. Suppose A is a complex matrix with only real eigen values. Show that A is similar to a matrix with only real entries.
20. Suppose T is normal. Prove that
    (a) T is self-adjoint if and only if its eigen values are real.
    (b) T is unitary if and only if its eigen values have absolute value 1.
21. T is positive if and only if its eigen values are nonnegative real numbers.
22. Show that any operator T is the sum of a self-adjoint operator and a skew-adjoint operator.
23. Let $\sim$ mean "is isomorphic to." Prove that $\sim$ is an equivalence relation on the class of vector spaces over F.
24. what is generalized eigenvectors of a linear operator T?

25. Let V and W be finite-dimensional vector spaces and T: V → W be an isomorphism. Let P be a subspace of V.
(a) Prove that T(P) is a subspace of W.
(b) Prove that dim(P) = dim(T(P))

26. Let T be a linear operator on an n-dimensional space V. Then show that The Characteristic and minimal polynomial for T have the same root.

27. Let V and W be vector spaces, and let T: V → W be linear and invertible. Then Show that T^{-1}: W → V is linear

28. Let $\beta = \{(2,1),(3,1)\}$ be an ordered basis for $\mathbb{R}^2$. Find the dual basis of β for $\mathbb{R}^2$.

29. Prove that the sum of two bilinear forms is a bilinear form.

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